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Plasma antenna system: dual-integral-equation method

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The study of the radiation pattern of the plasma column excited with the help of various types of sources seeks the help of Saddle point approach of steepest Descent method as done by pioneer Tamir *et al* (1962). Since then the problem of cylindrical plasma column has been solved for its infinite axial dimension Here problem is solved in semi-infinite space with the help of dual-integral-equation method. The radiation characteristic function gives two peaks which have the half beam width 3.6″ (homogeneous plasma column of radius 2.5 cm at $\phi = 89.59'$) and 2.7″ (Inhomogeneous plasma column of radius 2.5 cm at $\phi = 89.59'$). The general behaviour of the plot shows irregular variation except at $\phi = 89.59'$ and $269.59'$. The radiation does not happen to be vanishing in any direction.

1. Introduction

Tamir *et al* (1962, 63) studied the problem of excitation of plasma column with the help of sources and interpreted the radiation pattern due to the excitation of leaky waves on plasma columns. Excitation of leaky waves was studied by Goldstone *et al* (1959). But the significatn contribution came from Gupta *et al* (1967, 1970), Dhani Ram *et al* (1973, 1972), Ram Chandra *et al* (1974) and Joshi *et al* (1975) as far as the studies of excitation of cylindrical plasma column are concerned. The authors (1975a, 1975b, 1976a, 1976b, 1976c, 1976d) have also studied the cylindrical plasma excitation with the help of ring sources. All these studies seek help from the Saddle point approach to evaluate the radiation field. This approach is applicable for cylindrical geometries extending upto infinity in both the directions. But physical realization of such structures seems to be impossible. Therefore to make the theoretical studies closer to experimental feasibility, here a new approach for semi-infinite geometry is adopted. This will be one step up towards the experimental limitations than the previous one. A part of this study has been reported recently (Sharma *et al* 1976c).

2. Analysis and Conclusions

We have taken the geometry of the problem in cylindrical shaped and extending upto infinity in both the axial directions. In the plane $z = 0$, the source (ring of electric currents having radius a equal to that of plasma column). The plasma is assumed to be field in the $z < 0$ half space. It is assumed that the field is vanishing in $z > 0$ half space. It can be realized by assuming the presence of the conductor (of radius a) in $z > 0$ half space. Writing down the Maxwell's equation for the present case

$$\Delta \times \boldsymbol{E} = jw\mu_0 \boldsymbol{H} \quad \ldots \text{(1a)}$$

$$\Delta \times \boldsymbol{H} = -jw\epsilon_0\epsilon_p \boldsymbol{E} - \overline{M} \quad \ldots \text{(1b)}$$

Here μ_0, ϵ_0, ϵ_p and w happen to be free space permeability, permittivity, relative permittivity of plasma column and source frequency. Solving equs. 1 for E, one finds it to be nonlinear equation due to presence of M on the right hand side Here $\boldsymbol{M}$ is source term and being represented as

$$\boldsymbol{M} = \bar{\phi}\delta(\rho-a)\delta(z)$$

$\bar{\phi}$ stands for unit vector in ϕ-th direction of cylindrical coordinates (ρ, ϕ, z). δ can be recognized as Kronecker's delta function. The non linear eq. one gets, is following in cylindrical coordinates for exp $(im\phi)$ type azimuthal variation

$$\frac{\partial^2 E}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial E}{\partial \rho} + \left(w^2\mu_0\epsilon_0\epsilon_1 - \frac{m^2}{\rho^2}\right) E + \frac{\partial^2 E}{\partial z^2} = -jw\mu_0 \boldsymbol{M} \quad \ldots \text{(3)}$$

This equation can be made linear by taking Fourier transform as given by

$$E(\rho, z) = \int_{\infty}^{\infty} h(\rho, k)\exp(-jkz)dk$$

$$h(\rho, k) = \frac{1}{2\pi} \int_{\infty}^{\infty} E(\rho, z) \exp(jkz)dk$$

and applying proper boundary condition at $\rho = a + \Delta$ in place of $\rho = a$, $\Delta \to 0$ (Dhani Ram *et al* 1972) Now one can have eq. (3) in the following form

$$\frac{d^2h}{d\rho^2} + \frac{1}{\rho}\frac{dh}{d\rho} + \left(w^2\mu_0\epsilon_0\epsilon_p - k^2 - \frac{m^2}{\rho^2}\right) h = 0 \quad \ldots \text{(4)}$$

Solution to eq (4) will be obviously of the following form ($m = 1$).

$$h_1 = J_1(V_1\rho) \qquad 0 \leqslant \rho < a \quad \ldots \text{(5a)}$$

$$h_2 = AH_1^{(1)}(V_1\rho) \qquad a < \rho \quad \ldots \text{(5b)}$$

h happens to be Fourier's transform of E_ϕ.

$$V_1^2 = w^2\mu_0\epsilon_0\epsilon_p - k^2 \quad \text{and} \quad \alpha^2 = w^2\mu_0\epsilon_0\epsilon_p$$

w, μ_0, ϵ_0 and ϵ_p stand for source frequency, permeability permittivity (of free space) and relative permittivity of plasma column. α has to be complex

$E\phi_2$ will be given by

$$E\phi_2 = \left(\frac{1}{2\pi}\right)^{\frac{1}{2}} \int_{-\infty+ja}^{+\infty+ja} A(V_1)H^{(1)}(V_1\rho)\exp(-jkz)dk$$

where $-k < a < k$, the solution stands valid for $z > 0$ and $z < 0$ regions. But due to presence of half space $(z > 0)$ containing the conductor

$$\left(\frac{1}{2\pi}\right)^{\frac{1}{2}} \int_{-\infty+ja}^{+\infty+ja} A(V_1)H_1^{(1)}(V_1a)\, exp(-jkz)dk = 0 \quad \text{for} \quad z > 0 \qquad \dots (6)$$

with the help of jump-condition (Dhani Ram *et al*, 1972) one can have

$$\left(\frac{1}{2\pi}\right)^{\frac{1}{2}} \int_{-\infty+ja}^{+\infty+ja} A(V_1)V_1H_1^{(1)\prime}(V_1a)\exp(-jkz)dk$$

$$-\left(\frac{1}{2\pi}\right)^{\frac{1}{2}} \int_{-\infty+ja}^{+\infty+ja} V_1J_1'(V_1a)\exp(-jkz)dk = jw\mu_0 \qquad \dots (7)$$

where $H_1^{(1)\prime}$ and J_1' are respectively the differentiation of Hankel's founction of first kind and Bessel's function of first kind each of first order, with respect to their arguments. Now following Noble (1958) for dual integration equation approach one can come across the following expression just after little adjustments and calculations.

$$\left(\frac{1}{2\pi}\right)^{\frac{1}{2}} \int_{-\infty+ja}^{+\infty+ja} (\alpha-k)^{-\frac{1}{2}} V_1^2 A(V_1)H_1'(V_1a)\frac{\partial}{\partial z}\exp(-j\alpha z-jkz)\,dk = 0 \text{ for } z > 0 \qquad (8a)$$

$$\left(\frac{1}{2\pi}\right)^{\frac{1}{2}} \int_{\infty+ja}^{+\infty+ja} A(V_1)(\alpha-k)^{\frac{1}{2}}H_1^{(1)\prime}(V_1a)\exp(-jkz)dk$$

$$= N_+jw\mu_0 + \left(\frac{1}{2\pi}\right)^{\frac{1}{2}} \int (\alpha-k)^{\frac{1}{2}}J_1'(V_1a)\exp(-jkz)dk \quad z < 0 \qquad \dots (8b)$$

N_+ is in the notation of Noble (1958)

Taking Fourier's Inversion formula

$$A(V_1)H_1^{(1)\prime}(V_1a)(\alpha-k)^{\frac{1}{2}}$$

$$= -(2\pi)^{\frac{1}{2}}w\mu_0 \int_{-\infty}^{0} \frac{(\alpha+k)^{-\frac{1}{2}}}{(\alpha-k)}\exp(-j\alpha z)dz + (\alpha-k)^{\frac{1}{2}}J_1'(V_1a) \qquad \dots (9)$$

The complete solution in $z = 0$ plane for magnitude of $A(V_1)$ will be now given with the help of eq. (9)

$$A(\alpha) = \frac{J_1'(V_1a)}{H_1'(Va)} - \frac{(2\pi)^{\frac{1}{2}}w\mu_0}{\cos\phi(\alpha^2-k^2)\alpha}$$

for homogeneous plasma column

$$= \frac{J_1'(V_1a)}{H_1'(V_1a)} - \frac{(2\pi)^{\frac{1}{2}}w\mu_0}{\alpha(\alpha^2-k^2)\cos\phi} \qquad \dots (10)$$

for inhomogeneous plasma having in-homogeneity

$$\epsilon(\rho) = \epsilon \left[1- \frac{\rho}{a}\right]^2 \text{ and here } V_1^2 = w^2\mu_0\epsilon_0\epsilon(\rho)-k^2 \tag{11}$$

$$= \frac{J_1'(V_1a)}{H_1'(V_1a)} - \frac{(2\pi)^{\frac{1}{2}}w\mu_0}{\alpha(\alpha^2-k^2)\cos\phi}$$

for inhomogeneity of the type

$$\epsilon(z) = \epsilon \exp(+z/L) \text{ so } V_1^2 = w^2\mu_0\epsilon_0\epsilon(z)-k^2 \tag{12}$$

Equations 10, 11 and (12) are computed with the help of an IBM-1130 computer to calculate the magnitude of Ai, e the mangitude of the radiation field outside the ring source. The values for some parameters are $w_p = 3\times10^{10}$ rad sec^{-1}, $w = 2\pi\times10^{10}$ rad sec^{-1}, $z = 60.0$ cm, $L = 560.0$ cm $\rho = 1.5$ cm.

The general field characteristics are plotted in figure.1 which shows irregular

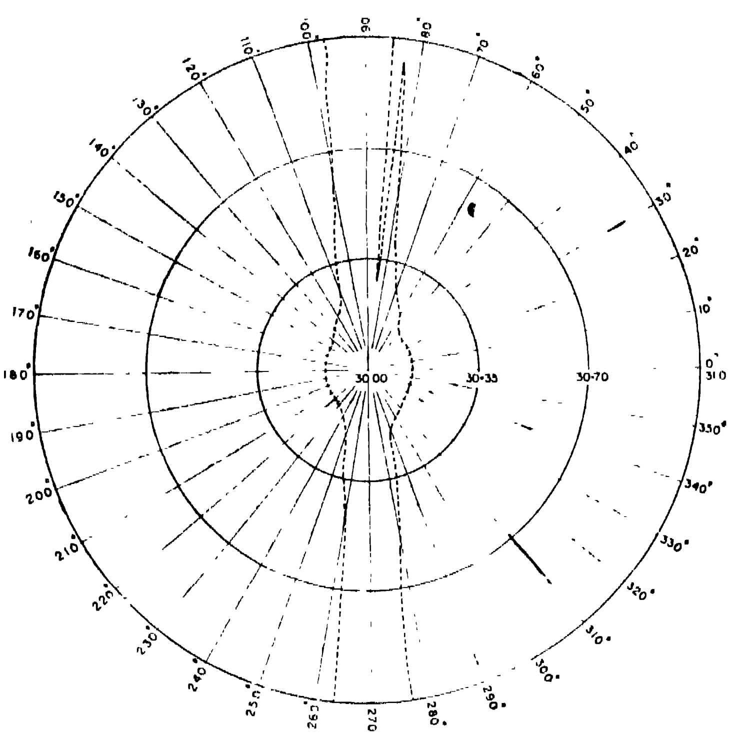

Fig. 1. The variation of the field amplitude for $a = 2.5$ cm. The discontinuity in graph shows the larger field amplitude at those angles. The plasma has exp (z/L) inhomogeneity.

change in the amplitude of the field with different ϕ. Still no direction is observed having negligible radiation. In the calculation we have got two intense sharp peaks occuring very near to $\phi = 90^\circ$ and 270°. The sharp singularity is therefore at above angles avoided and in the near vicinity of above angles the field calculations are done. The HBW comes out to be of the order of $3.6''$ for homogeneous plasma and $1.8''$ for inhomogeneous plasma $\left[1-\left(\frac{\rho}{a}\right)^2\right]$ near $\phi = 90^\circ$ for $a =$ 2.5 cm. The HBW happens to be same for these peaks occurring near $\phi = 270^\circ$. Figure 2 shows the sharpness of the peaks. The strength of the radiation magnitude in the present case happens to be stronger than that of calculated by Sharma *et al* (1975a, 1975b). As far as this type of behaviour of the semi space filled plasma is considered, it does not resemble with the studies carried out with the help of saddle point integration method. Here one finds sufficient reason to continue the study for it is a bridge between infinite and finite geometry studies. Our further attention will be to get more clean and simble expression for the amplitude of the radiation field. This will be published shortly.

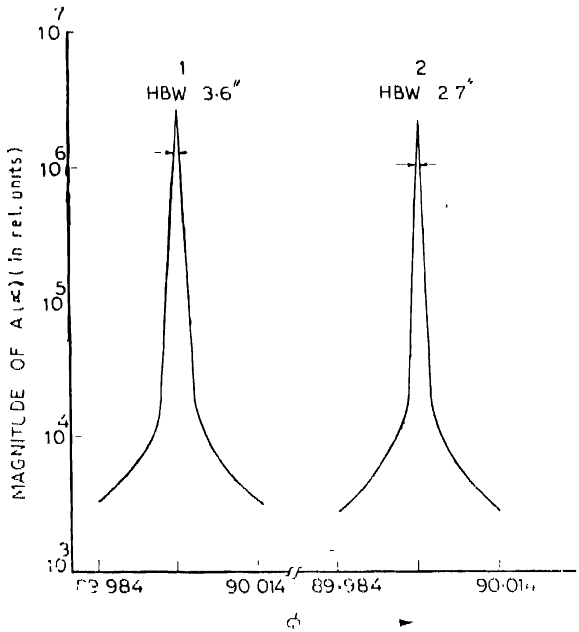

Fig. 2. For $a = 2.5$ the peak of the radiation field due to the ring of electric currents fo (*i*) Homogeneous plasma column (*ii*) Inhomogeneous plasma column. $\varepsilon(z) = \varepsilon \exp(Z/L)$ type inhomogeneity.

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References

Dhani Ram & Verma J. S. 1972. *Ind. J. Pure Appl. Phys.* **10** 716.

Dhani Ram & Verma J S. 1973. *Int J. Electron* **34**, 831.

Goldstono L. O & Oliner A. A. 1959. *I R.E Trans. Antenna Propag. A.P.* **7**, 307.

Gupta K. C. & Singh A 1967 *Int. J Electron* **23**, 133.

Gupta K. C. 1970. *Int. J. Electron*, **29**, 45.

Joshi N. K. & Verma J. S 1975 *Rad Sc (U.S. I)* **2**, 187.

Noble B. 1958 *Wiener--Hopf Technique* (pergamon Press, London) 58.

Ram Chandra & Verma J. S 1974 *Ind. J Pure Appl. Phys.* **12**, 192

Sharma Subhash Chandra, Ram Chandra & Verma J. S. 1975a. *Ind. J. Pure Appl. Physics*, **13**, 829.

Sharma Subhash Chandra & Verma J S 1975b *Ind J. Rad. Space Phys.* **4**, 282

Sharma Subhash Chandra, Ram Chandra & Verma J. S. 1976a, presented in the *Symposium on Solar Planetary Physics* held at PRL, Ahmedabad during Jan. 20-24 76.

Sharma Subhash Chandra, Ram Chandra & Verma J S. 1976b, to appear in JIETE in Aug. 1976 issue.

Sharma Subhash Chandsa & Verma J. S. 1976c. Accepted for publication in Ind. J. Phys

Sharma Subhash Chandra & Verma J. S. 1976d, to appear in Ind. J. Phys.

Sharma Subhash Chandra & Verma J. S. 1976e. Communicated to Ind. J Phys.

Subhash Chandra Sharma & Verma J. S. 1976f, presented in the symposium on plasma physics and its applications held at PRL, Ahmedabad during Aug. 9-14 '76.

Tamir T. & Oliner A A. 1962 *IRE Trans. Antenna Propag. AP*-10, 55.

Tamir T. & Oliner A. A. 1963. *Proc. IEE*, **110**, 325.